Boundary Element Methods for the Prediction of Sheet and Developed Tip Vortex Cavitation

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Abstract

Recent applications of boundary element methods to predict sheet or developed tip vortex cavitation on lifting bodies, within the framework of nonlinear cavity theory, are summarized. A Dirichlet type of boundary condition is applied on the cavity surface, while a Neumann type of boundary condition is applied on the wetted (non-cavitating) body surface. The shape of the cavity is determined via an iterative technique until both, the constant pressure condition and the flow tangency condition, are satisfied on the cavity surface. 2-D, 3-D hydrofoils, submerged marine propellers in non-uniform inflow, as well as surface-piercing propellers are considered. Some comparisons with experiments are presented, and future challenges are outlined.

1 INTRODUCTION

Sheet cavitation, the type of cavitation which is characterized by a “continuous” liquid/vapor interface which is “attached” to the body, can often occur during the operation of hydraulic or hydrodynamic devices, e.g. hydrofoils, pumps, marine propulsors. Despite its undesirable nature, some sheet (or other types of) cavitation often has to be accepted, so that the efficiency of these devices is not excessively low. The appearance of sheet cavitation is becoming more common in recent years as the lifting surface loads and the flow speeds increase. Its prediction therefore becomes a very crucial aspect during the hydrodynamic design or assessment stage.

Cavitating or free-streamline flows were first addressed in nonlinear theory via the hodograph technique as introduced by Helmholtz, Kirchoff and Levi-Civita (Birkhoff and Zarantonello, 1957). The cavity surface in steady flow was taken as a steamline with constant pressure (thus, constant velocity).

The linearized cavity theory was introduced by (Tulin, 1953) and became quickly very popular, as proven by the vast amount of publications which made use of it. An extended list of related

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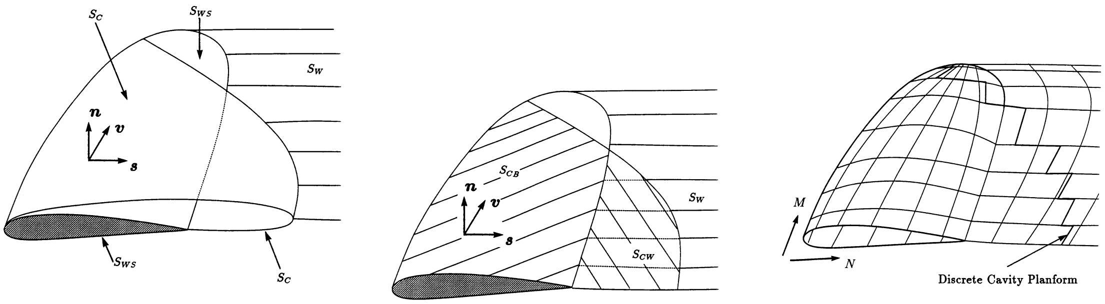

Figure 1: Definition of the "exact", the approximate, and the discretized 3-D cavity and foil surface.

publications may be found in (Tulin and Hsu, 1980) or (Kinnas, 1991).

In the present work boundary element methods for the prediction of sheet and developed tip vortex cavitation, within nonlinear cavity theory, will be summarized, and some comparisons with experiments will be presented.

2 3-D HYDROFOIL

2.1 Formulation

Consider now a 3-D hydrofoil which is subject to a uniform inflow $\mathbf{U}_\infty$ as shown in Fig. 1. The cavity surface is denoted with S_C, the wetted hydrofoil surface with S_{WS}, and the trailing wake surface with S_W. The total flow velocity field $\mathbf{q}(x,y,z)$, can be written in terms of the perturbation potential, $\phi(x,y,z)$, as follows:

$$\mathbf{q}(x,y,z) = \mathbf{U}_\infty + \nabla\phi(x,y,z). \tag{1}$$

In the next four sections the necessary equations and conditions for determining $\phi(x,y,z)$, as well as the cavity planform and shape are outlined. Only the non-linear cavity solution is described. References on linearized approaches have been given in the introduction.

2.1.1 The Green's formula

As in the case of 2-D hydrofoil Green's third identity renders the following integral equation for $\phi(x,y,z)$:

$$2\pi\phi = \int_{S_{WS}\cup S_C}\left[\phi\frac{\partial G}{\partial n} - G\frac{\partial \phi}{\partial n}\right]dS + \int_{S_W}\Delta\phi_W\frac{\partial G}{\partial n}dS \tag{2}$$

$\mathbf{n}$ is the unit vector normal to the foil wetted surface, the cavity surface or the wake surface; $\Delta\phi$ is the potential jump across the wake sheet; $G = 1/R$ is the Green's function, where R is the distance between a point P and the point of integration along the foil and cavity surface.

$$\frac{\partial \phi}{\partial n} = -\mathbf{U}_\infty \cdot \mathbf{n}; \quad \text{on } S_{WS} \tag{3}$$

In the case of partial cavitation the trailing wake, S_W, is treated the same way as in the case of fully wetted flows (Morino and Kuo, 1974). In other words $\Delta\phi_W$ is only a function of the spanwise location y, given as:

$$\Delta\phi_W(y) = \phi_T^+(y) - \phi_T^-(y) = \Gamma(y) \tag{4}$$

where $\phi_T^+(y)$ and $\phi_T^-(y)$ are the values of the potential at the upper (suction side) and lower (pressure side) foil trailing edge, respectively. The difference in those potentials is also equal to the circulation Γ at a spanwise location y. In recent years we apply the iterative pressure Kutta condition at the trailing edge (Kinnas and Hsin, 1992), in which $\Delta\phi_W$ is determined from the requirement that the pressure jump across the trailing edge is equal to zero everywhere along the span.

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3 CONCLUSIONS

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4 ACKNOWLEDGMENT

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